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Physical Conditions in the Primitive Solar Nebula

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Abstract

Models of the primitive solar nebula have been constructed. A model is required to be in centrifugal equilibrium radially in the plane of the disk and in hydrostatic equilibrium perpendicular to the plane of the disk. The distribution of angular momentum per unit mass in the initial model is that appropriate to a fragment of a collapsing interstellar gas cloud.

For a reasonable distribution of the angular momentum per unit mass, there are two solutions with centrifugal equilibrium. One solution has a very flat surface density, which is unstable against many perturbations, and presumably forms a double star system. The other solution is much more axially condensed and is taken to be an approximate model of the primitive solar nebula. This model is formed as a result of a dynamical collapse process which produces initial interior temperatures in the vicinity of 10^4 °K. The collapse causes initial turbulence, and there is a resulting redistribution of angular momentum as a result of turbulent viscosity. This has been taken into account in an approximate way and the resulting redistribution of surface density has been calculated.

The nebula cools at constant central pressure. In the range 2000-5000 ^{o}K the opacity is small and the nebula is in radiative equilibrium. At lower temperatures the opacity is greatly increased due to molecules like H_2O and to condensed particles, mainly iron and silicate grains. This produces thermally-driven convection out to a nebular radius of a few astronomical units. In this region there is renewed turbulent viscosity, which leads to outward transport of angular momentum and inward net transport of mass to form the sun.

The accumulation of small particles into larger bodies always takes place in the presence of gas and is assisted by turbulence and acceleration by electric fields. Lightning flashes may play a role in the formation of chondrules. Continued convective transport through a range of temperatures must play an important role in the chemistry of the accumulating material. The relation of these processes to the range of conditions presented in the nebular model is discussed.

Several years ago I presented an analysis of the conditions in which an interstellar cloud could become unstable against gravitational collapse [1]. For a cloud of 1000 solar masses, an initial density of about 1000 hydrogen atoms per cubic centimeter is required. High initial densities of this kind may be produced if the outer regions of a neutral hydrogen cloud are ionized by the ultraviolet radiation from an O or B star, thus creating the necessary high surface pressure for the compression to high density.

It is to be expected that such a cloud will be turbulent and that its collapse will be roughly isothermal. The resulting density fluctuations should initiate fragmentation in the cloud. In a recent paper I concluded that the turbulent component of the internal angular momentum of a fragment should be comparable to that associated with an initial corotation of the interstellar cloud with its orbital motion about the center of the galaxy [2].

The spherical collapse of such a fragment would form an object on the high luminosity "Hayashi" track in the Hertzsprung-Russell diagram which characterizes stellar evolution. However, considerations of the conservation of angular momentum in the collapsing fragment indicate that it should form a rotating flat disk with a radius of several tens of astronomical units, much larger than a star would possess on the Hayashi track [1,3]. It has been the objective of the present work to discover how such a disk might dissipate to form the sun and the solar system.

In order to relate the problem to the interstellar cloud conditions, the cloud fragments were assumed to be uniformly-rotating spheres. The equators of these spheres were assumed to have conserved their local angular momentum from the initial cloud corotation condition with its angular velocity of 10^{-15} radians/second. The density was assumed to vary linearly with radius from a central value to a surface value. The specific models used in the present calculations have two solar masses. In one sphere the density was uniform

("uniform sphere") and in the other sphere the density fell linearly from the central value to zero at the surface ("linear sphere").

Each sphere was divided into 50 cylindrical zones concentric about the axis of rotation, and the mass and angular momentum of each zone was calculated. The mass was then considered to have collapsed into a thin flat disk, and it was required that the mass in the disk be everywhere in centrifugal equilibrium with respect to the gravitational forces at that point in the disk. Radial pressure gradients were neglected since I was interested in the case in which thermal energies in the gas would be much less than the bulk rotational kinetic energies.

Gravitational potentials were calculated by the technique of a super-position of concentric spheroidal shells of varying density in the limit of zero eccentricity [4-8]. With some trial mass distribution in the disk, the surface density in each zone was varied and the changes in angular momentum per unit mass required for circular motion for all the other zones were determined. A matrix was then inverted to determine what perturbations to introduce into the surface densities of all the zones so that the angular momentum required for circular motion in the model would approach the assigned values.

It was discovered that each of the two original spheres had two different surface density distributions for which it was in centrifugal equilibrium. The four solutions are shown in Figure 1. The flat solution for the uniform sphere is known classically [9]; it is in uniform rotation. In addition there is an axially-condensed solution. Mestel [8] has discussed such axially-condensed solutions, which he shows can be produced by slight perturbations in the mass distribution of a uniform sphere. However, it has been shown by the present numerical technique that the same mass and angular momentum distribution is consistent with each solution. In the axially-condensed solution the angular velocity varies roughly inversely as the radial distance, so that there is a large amount of shear in the solution.

The flattened linear sphere exhibits a similar behavior, as shown in Figure 1. There is a nearly-flat solution, which is nearly in uniform rotation, and which has a very sharp edge. The axially-condensed solution is more strikingly so. The irregularity of this solution near the axis is an artifact of the finite zoning and has no physical significance. This solution also has a condensed ring near the outer edge, which will be discussed in more detail below.

I believe that the existence of these two solutions has great cosmogonic significance. Hunter [10] has examined the stability of the classical uniformly-rotating flat disk. He has found it to be unstable against both radial and non-radial perturbations. (Professor Gold has informed me that he and Bondi have found a very limited range of parameters in which this appears not to be true.) K.H. Prendergast (private communication) has shown numerically that a flat galactic disk with superposed random velocities of the individual mass points deforms into a corotating bar. It appears likely that the corotating disk will deform in a similar manner and will subsequently form a close pair of binary stars. No further calculations have been carried out with these flat solutions. Whether a collapsing interstellar cloud fragment forms a flat disk or an axially-condensed disk must depend upon subtle features of the dynamics of the collapse.

I refer to the axially-condensed protostellar disks as "stellisks". I assume that they are stable against perturbations, but no analysis of this point has been made.

When a stellisk is formed as a result of the dynamical collapse process, the gas will overshoot the position of centrifugal equilibrium, thus inducing a state of initial turbulence in the disk. The energy input into the turbulence arises from the released gravitational potential energy. Since there is no continuing source of energy input into the turbulence, it is short-lived. The largest eddy motions are broken into smaller motions after the gas has moved through a mixing length, which takes a small fraction of an orbital period.

I have estimated the angular momentum transfer between adjacent zones due to turbulent viscosity, using data appropriate to the stellisk models of Figure 1. It turned out that the angular momentum transfer would be sufficient to make adjacent zones corotate if they did not change their radial positions. In fact, the angular momentum transfer would spread the zones apart and increase the shear between them. The situation was treated very crudely by taking the inner five zones for a stellisk model and redistributing the angular momentum so that they would corotate at their given positions. This procedure was then applied to zones 2 to 6, 3 to 7, and so on in the model until the outer edge was reached. The models were then again relaxed to centrifugal equilibrium.

The justification for this procedure lies in the fact that the center of a cloud fragment will collapse faster than the surface layers, so that the turbulent redistribution of angular momentum will be completed near the center before the outer parts have finished collapsing.

The model of the uniform sphere stellisk after dissipation of initial turbulence is shown in Figure 2; it is compared in the figure with the initial model of Figure 1. It may be seen that a slight further net central condensation of mass has occurred, except near the outer edge. A prominent outer condensed ring has formed, and a slight second ring-like perturbation is also present.

A similar behavior for the linear sphere is shown in Figure 3. In this case two prominent condensed outer rings are present.

It may be deduced from these figures that condensed rings will have an increasing tendency to form as the relative mass fraction in the outer layers is reduced and as the gradient of the angular momentum per unit mass in the outer layers is increased. The reality of the rings was tested by compressing and smoothing the outer zones of the linear sphere stellisk and relaxing again to centrifugal equilibrium. Again the two condensed rings appeared, but the outer one centered on a different zone. Thus the reality of the ring structure was demonstrated, but the uniqueness of the structure remains an open question. Calculations with a finer zoning mesh are needed for a further investigation of this question.

It seems likely to me that these rings will be unstable against non-radial perturbations, and that they will deform to form separate disk-like condensations orbiting about the central disk. Such sub-disks seem likely precursors of the giant planets of the solar system.

The collapse of the interstellar cloud fragment will, in the late stages, lead to adiabatic heating of the inner parts of the stellisk to 10^4 °K or higher. There will be an initial period of rapid radiative cooling. Further dissipation of the disk will depend on the operation of turbulent viscosity to transport angular momentum outwards. The only apparent energy input source for such turbulence would be thermally-driven convection. This would require superadiabatic temperature gradients to exist perpendicular to the plane of the disk. Hence the structure of the disk perpendicular to the plane was investigated with the simplifying assumption that any column of the disk could be considered part of an infinite plane of matter having the same local conditions.

It is instructive to examine the equations for the structure of the disk perpendicular to the plane under the simplifying assumption that the disk is isothermal [8]. Such a condition would hold when there is negligible opacity in the disk. We asume the perfect gas law:

$$P = \frac{N_o k T \rho}{\mu} \tag{1}$$

The gravitational potential φ can be obtained from Poisson's equation

$$\frac{d^2\varphi}{dz^2} = -4\pi G\rho \quad , \tag{2}$$

where z is the distance perpendicular to the plane measured from its center. The equation of hydrostatic equilibrium is

$$\frac{1}{\rho}\frac{dP}{dz} = \frac{N_o kT}{\mu\rho}\frac{d\rho}{dz} = \frac{d\varphi}{dz} \tag{3}$$

Let us define the particle and total surface densities as

$$\frac{\sigma_z}{2} = \int_o^z \rho dz \tag{4}$$

$$\frac{\sigma_s}{2} = \int_o^\infty \rho dz \tag{5}$$

The solution of (2) and (3) is

$$\rho = \rho_c\left[1-\left(\frac{\sigma_z}{\sigma_s}\right)^2\right] = \rho_c \ \mathrm{sech}^2\left(\frac{z}{z_u}\right) , \tag{6}$$

where ρ_c is the density at the center of the disk and z_u is the semi-thickness which the disk would have if it were uniform with the central density. The partial surface density σ_z is

$$\sigma_z = \sigma_s \tanh\left(\frac{z}{z_u}\right) . \tag{7}$$

Denoting the central pressure by P_c, we have

$$\rho_c = \frac{\pi G \mu \sigma_s^2}{2 N_o k T} \tag{8}$$

$$P_c = \frac{\pi G \sigma_s^2}{2} \tag{9}$$

$$z_u = \frac{N_o k T}{\pi \mu G \sigma_s} . \tag{10}$$

It should be noted from equation (9) that the central pressure is independent of the temperature; hence it remains constant as the disk cools. However, it depends strongly on the surface density in the disk, ranging downwards from $\sim 10^2$ atmospheres near the center of the stellisks of Figure 3. The semithickness parameter z_u is proportional to the temperature and inversely proportional to the surface density, as may be seen from equation (10). Thus the semithickness decreases as the disk cools, but it is greater near the edge of the disk where σ_s is small than near the center where σ_s is large. The latter feature is of prime importance for considerations of the chemical accumulation of matter in the disk.

In a more realistic disk there is a finite opacity. Hence a temperature gradient must be set up which will transport energy to the surfaces of the disk, where it can be radiated away. Thus an additional equation for the temperature gradient is needed for the determination of the disk structure. This situation is analogous to the equations of stellar structure which have been extensively studied. If energy transport is by radiative transfer, then

$$\frac{dT}{dz} = \frac{3}{4ac} \frac{K\rho}{T^3} L , \tag{11}$$

where a is the radiation constant, c is the velocity of light, K is the opacity, and L is the energy transport in erg/cm^2 sec. If the opacity is sufficiently high, then the gas becomes unstable against convective energy transport, and the temperature gradient is approximately adiabatic:

$$\frac{dT}{dz} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{P} \frac{dP}{dz} , \tag{12}$$

where γ is the ratio of specific heats. One must use whichever of equations (11) and (12) gives the smaller temperature gradient.

As explained above, angular momentum transport in the solar nebula and dissipation to form the sun depends critically on the presence of convection. At temperatures in the general vicinity of 3000 °K and pressures of less than a few atmospheres, the opacity is rather low and convection should not be present. However, at temperatures near 1000 °K, chemical condensation of iron and silicate particles has occurred and the molecular opacity of H_2O, NH_3, and CH_4 is important.

As far as the particles are concerned, the main contribution to opacity comes from the metallic iron grains. Judging from electron microscope pictures of very primitive meteorites (E. Anders, private communication), the iron grains should have sizes in the submicron range, much less than the wavelengths of the radiation in the nebula, and the opacity can be calculated without ambiguity (I am indebted to J.M. Greenberg for providing me with the necessary data).

I have calculated some typical disk structures using solid particle opacities only. The typical result is that no convection is present for small values of σ_s, but between surface densities of 10^5 and 10^6 grams per square centimeter convection sets in. I am currently preparing to do the calculation with the addition of molecular opacities, and I expect the threshold for convection to be lowered, perhaps to between 10^4 and 10^5 grams per square centimeter.

It may be noted that the surface densities exceed the threshold for convection in the stellisks shown in Figures 2 and 3 out to a few astronomical units. Hence this is the range of material in the stellisks which may be expected to dissipate to form the sun. The amount of material in these models out to the indicated distance is of the order of a solar mass.

The more realistic structure of the disk remains close to that of the isothermal case. Most of the temperature drop occurs close to the surface. If the surface density is much above the threshold for convection, then the disk is convective everywhere except near the photosphere, where it becomes in radiative equilibrium. However, near the threshold for convection, the convective transport occurs only near the surface, and the major part of the disk near the center is in radiative equilibrium. Since this is the region in which chemical accumulation of meteorites probably occurred, this complicated radiative-convective structure may be of primary importance in meteorite studies.

Chemically condensed particles in the submicron range will remain effectively suspended in the gas of the disk, since gravitational potential gradients are very small. The presence of this gas is extremely important for accumulation processes, since it assures that particles will collide very gently and can hold together if their surfaces have affinities for each other.

Processes important in the accumulation of meteoritic material include:

1. Gravitational settling. As remarked above, this is a very slow process owing to the small values of the gravitational potential gradients.

2. Electrostatic acceleration. Natural radioactivity will produce ionization in the nebula, and turbulence will tend to separate charges and establish electrostatic potential gradients [11]. Resulting lightning discharges may play a role in the melting and formation of chondrules [11,12]. Particles with different charge to mass ratios will suffer variable accelerations, which leads to collisions among them.

3. Turbulence. The turbulence is characterized by a shearing motion in the gas, which will bring together particles slightly displaced perpendicular to the direction of the shear.

4. Radial pressure gradients. Although the gas pressure gradient in the radial direction in the plane of the disk has been ignored here, nevertheless it will have a finite value. Hence the gas will rotate at slightly less than the Keplerian orbital velocity, but the particles will try to have Keplerian velocities. Friction with the gas will lead to a slow drift of particles inward in the nebula, the rate of which varies with the mass to projected surface area ratio.

A feature which greatly complicates the accumulation process is the necessity for some sort of chemical affinity between the surface layers of the colliding particles. Presumably this varies with temperature and composition.

The largest turbulent eddies in the thermally-driven convecting gases will have dimensions comparable to the scale height z_u. For the temperatures of interest, this height is comparable to or not too much less than the radial distance in the disk (thus indicating that the approximation of decoupling the radial and perpendicular properties of the disk here is only a rough approximation). Thus the large scale of the turbulent eddies will have two important effects on the accumulation process:

1. Chemically condensed materials will be subject to extensive radial transport in the nebula during the accumulation process. This can introduce chemical inhomogeneities in the accumulating material.

2. The turbulent viscosity will be very large, leading to a rapid dissipation of the inner solar nebula in a time not too many orders of magnitude greater than the orbital periods. A general dissipation time $\sim 10^3$ years is thus crudely indicated. The accumulation of solids into planetary bodies is thus inefficent, and it is not surprising that the planets contain only $\sim 10^{-2}$ of the chemically condensed fraction of the inner nebula. As a result of the rapid release of gravitational potential energy in the surface layers of a planet like the earth, temperatures in the outer layers of several thousand degrees will be produced, leading to volatilization and loss to the solar nebula of many of the solid constituents. Volatile elements will be retained only near the center of the earth and in small bodies (including meteorites) which fall on the earth later.

After the sun has formed, it will be a "T Tauri" star. Such stars are observed to emit mass at a rapid rate, typically a solar mass per million years. This mass loss is presumably a form of enhanced solar wind having some 10^7 times the flux of the present solar wind. This will sweep away the primitive atmospheres which would be captured by the inner planets from the nebular gases, and it will also sweep away the residual nonconvective gas in the outer part of the solar nebula.

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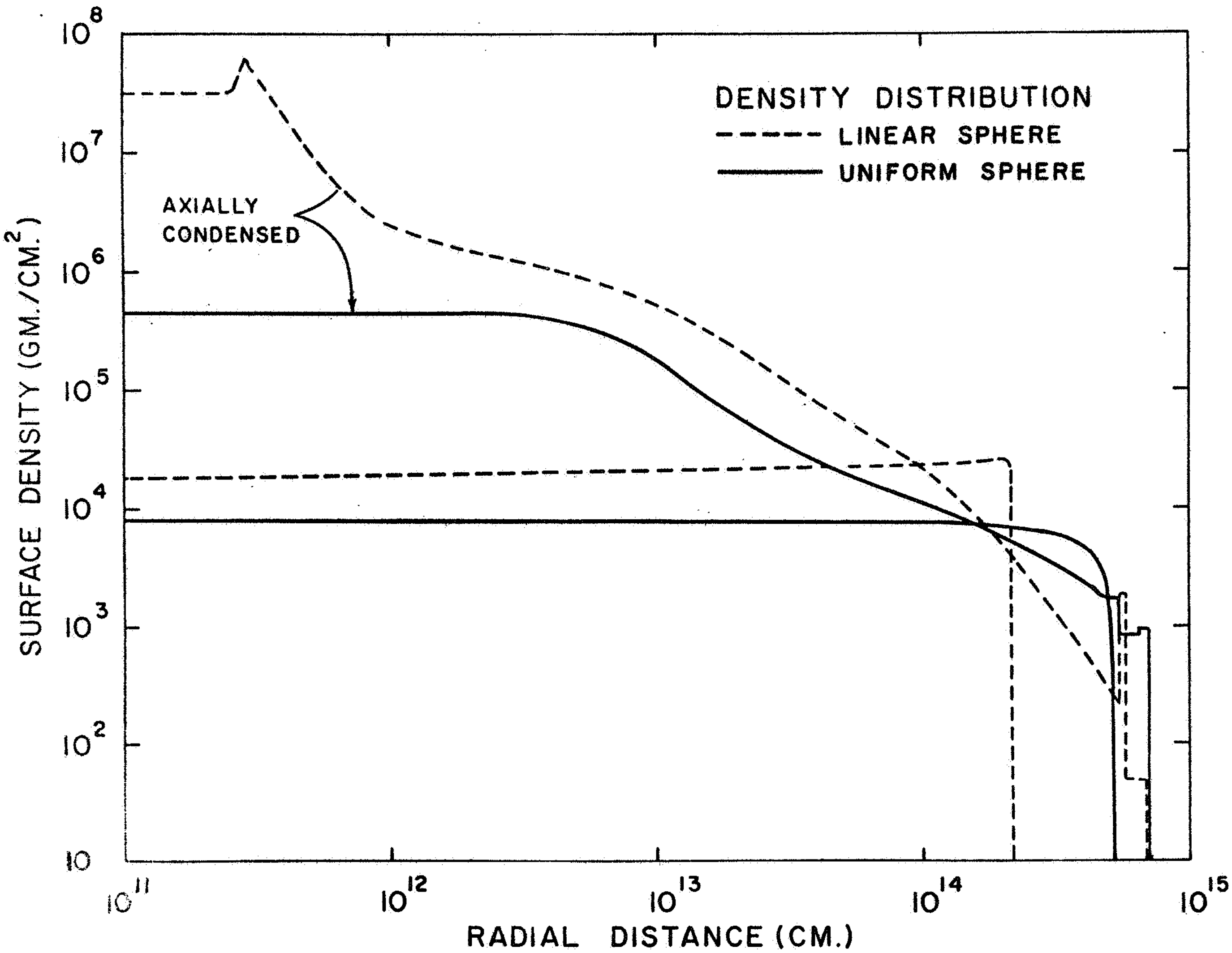

Figure 1. Surface density distributions for centrifugal equilibrium of disks formed from collapse of uniform and linear spheres.

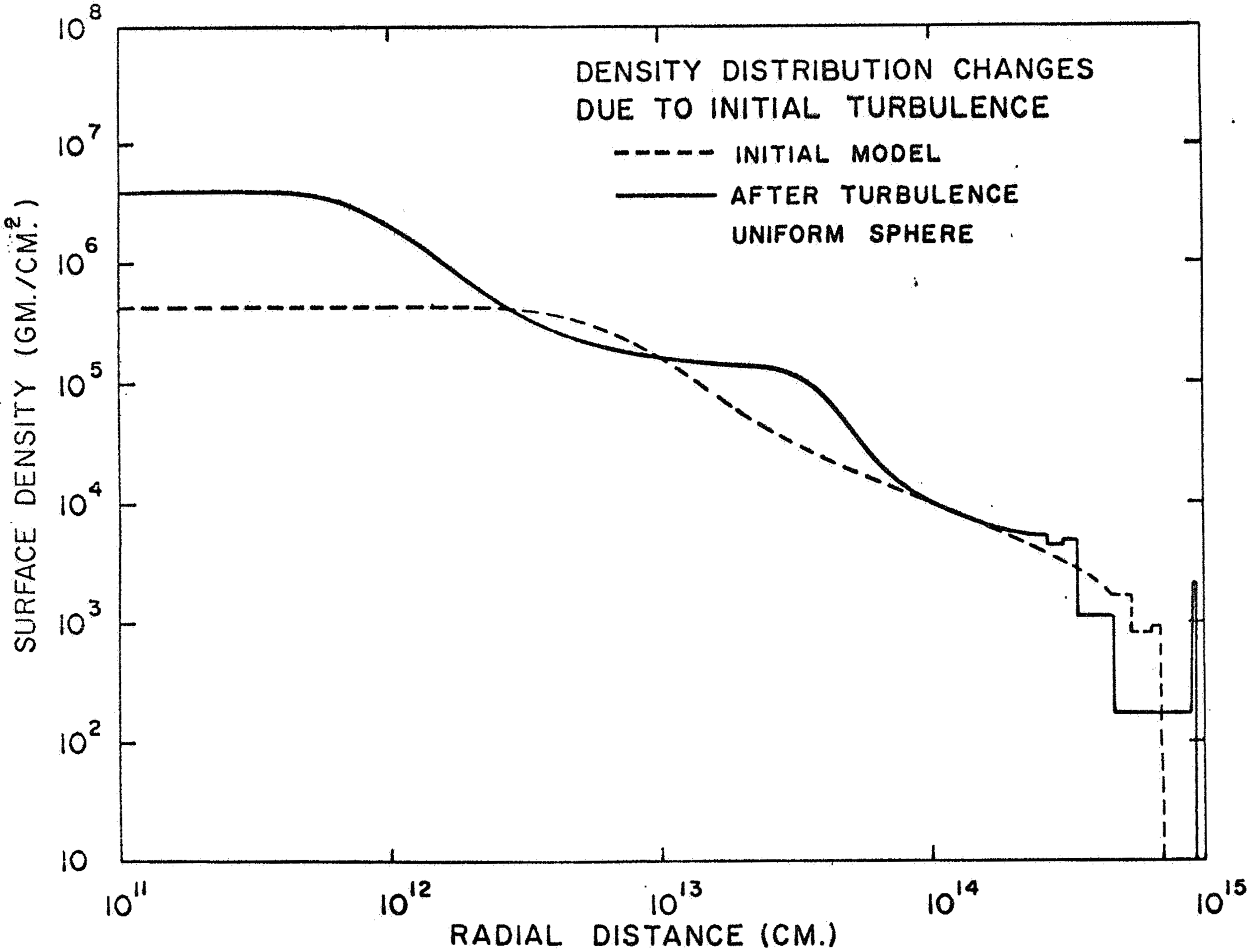

Figure 2. Changes in surface density distributions in the uniform sphere stellisk after dissipation of initial turbulence.

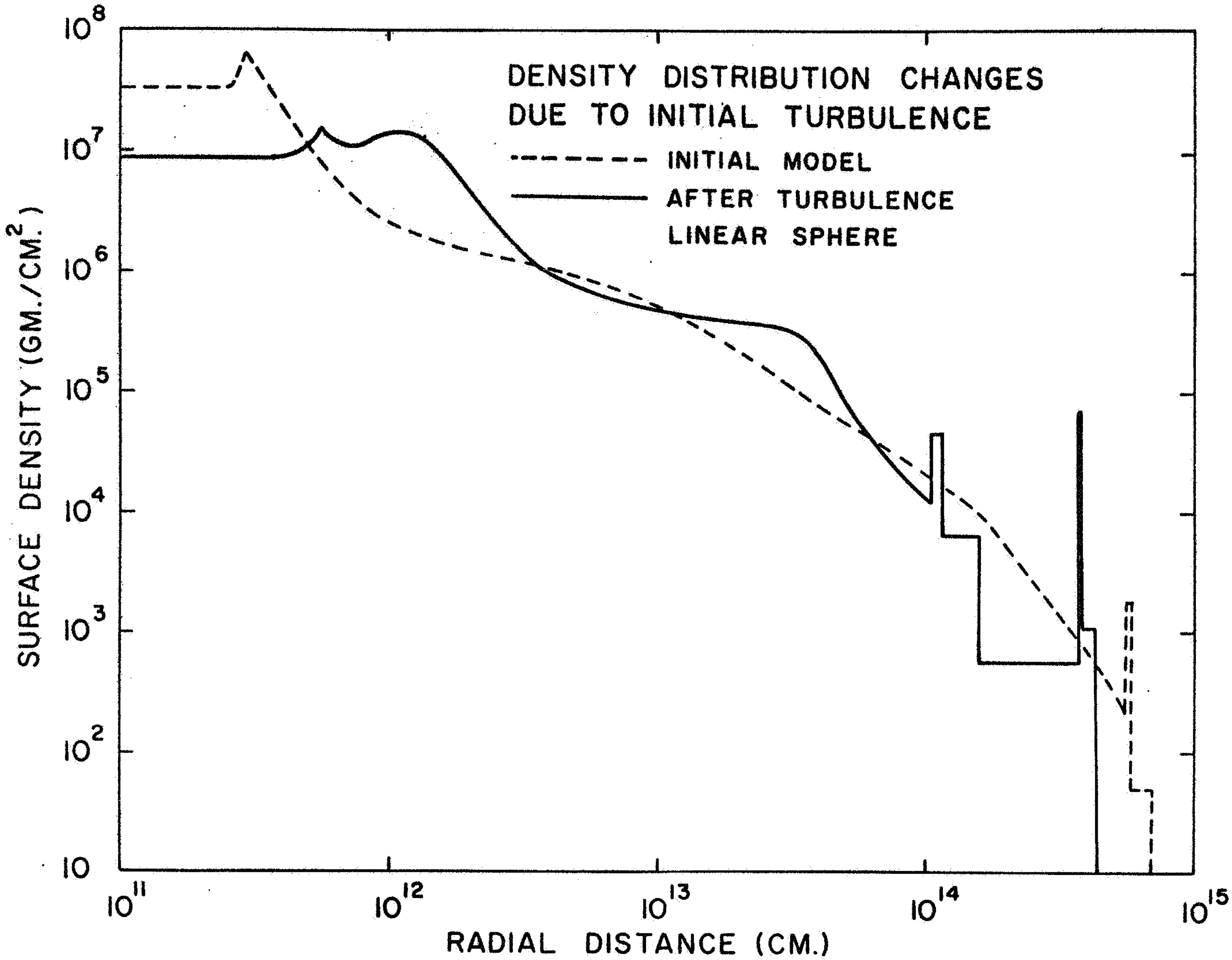

Figure 3. Changes in surface density distributions in the linear sphere stellisk after dissipation of initial turbulence.